LEARNING THROUGH SPORTS™

# Mathematics:
# Operations with Whole Numbers

The following section of this customized textbook includes material from these skill areas:

**Skill Description**

2089: add three one-digit numbers

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2096: estimate addition solutions

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2100: solve addition problems using currency

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2101: solve addition problems with decimals

4.MD.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

2103: solve addition problems with fractions

4.NF.3.d: Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.3.a: Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

2104: solve addition problems with single-digit numbers

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2106: solve addition problems with whole numbers

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2109: explore concepts of number lines

4.MD.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

2111: use the commutative property of addition

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2114: use the zero property of addition

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2122: apply division in real-world situations

4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

2130: solve division problems with whole numbers with and without remainders

4.NBT.6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

2139: estimate solutions to multiplication problems

4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

2140: explain inverse relationship of division and multiplication

4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

2144: solve multiplication problems with whole numbers

4.NBT.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

4.OA.1: Interpret a multiplication equation as a comparison, e.g., interpret 35 is 5 times as many as 7 and 7 times asmany as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

2159: apply subtraction in real-world situations

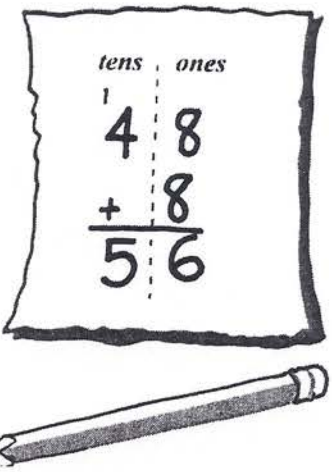
4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2170: solve subtraction problems with whole numbers

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

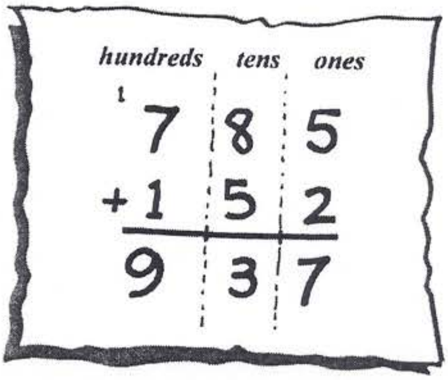
# Addition with Carrying (or Renaming)

When addition results in a sum greater than 9 in any place, any amount over 10 is ***carried*** to the next·place.



In the ones place of this example, $8 + 8 = 16$.

The 6 is written in the **ones** place, and the rest of the amount (1 ten) is ***carried*** over to the **tens** place. That extra ten is added to the other tens in the column. ***Renaming*** and ***regrouping*** are other terms used for this process.

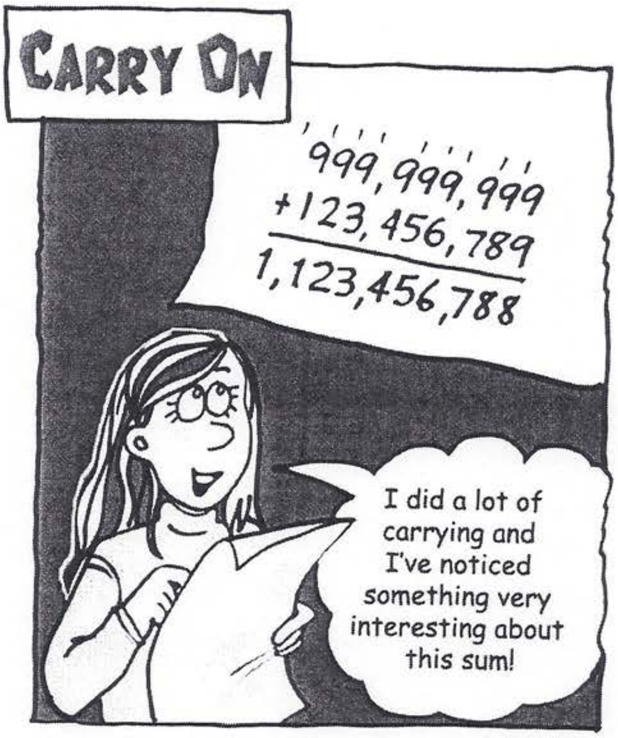


In the example, $8 + 5$ equals 13. Since these digits are in the tens place, the sum of these digits has a value of 13 tens.

The amount of 13 tens is ***renamed*** as **3 tens and 1 hundred.** The 3 is written in the **tens** place, and the 1 (value of 100) is added to the other amounts in the **hundreds** column.

Several addends lined up beneath each other form a column. This is called ***column addition.***

$$\begin{array}{r} 17 \\ 88 \\ 42 \\ 90 \\ 5 \\ +\,66 \\ \hline 308 \end{array}$$



# Estimating

To **estimate** means to make a reasonable guess. Estimation is a good tool to use when you do not need to end up with a precise count or answer. Rounding is helpful when you estimate.

A school full of kids went to the circus. Each kid dropped 88 peanut shells on the floor. There were 11 kids in each row, and 32 rows in the section. How many peanut shells were dropped on the floor in that section?

Round the 88 peanut shells to 100. Round the 11 kids in a row to 10. Round the 32 rows to 30. Multiply 100 x 10 x 30. The estimated number of shells is 30,000!

One class of 21 students really enjoyed the food at the circus. Each student wanted 2 bags of peanuts, 2 hot dogs, 1 cotton candy, 1 large drink, and 1 ice cream bar. The students had a total of $275.00 between them. Is that enough to pay for their lunch?

## Menu

| | |
|---|---|
| **Hamburgers** | $3.65 |
| **Hot Dogs** | $2.75 |
| **Fries** | $1.75 |
| **Ice Cream Bar** | $2.10 |
| **Peanuts** | $ .90 |
| **Cotton Candy** | $1.85 |
| **Popcorn** | $2.00 |
| **Drinks** | |
| **Small** | $ .75 |
| **Large** | $1.25 |

Round the hot dog price to $3, ($6 for two), the peanuts to $1 ($2 for two), the cotton candy to $2, the drink to $1, and the ice cream bar to $2. This is a total of $13 per person. Round the 21 students to 20. 20 x 13 is $260.00. $275 should be enough.

# Money

**Get Sharp Tip #17**
When you do calculations with money, always round off amounts to the nearest hundred (cents).

Decimals are used to write amounts of money.
Money is shown in decimal numbers to the hundredth place.

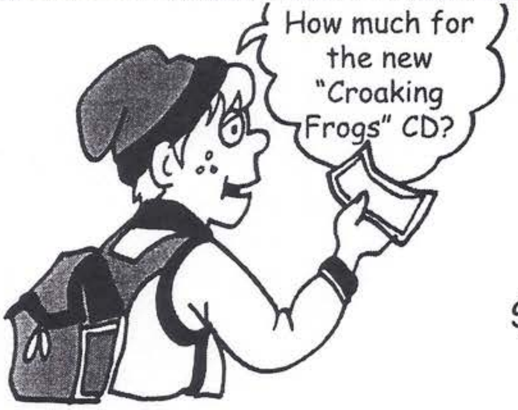
In a money amount, the decimal point is placed after the dollars (to the right of the one dollars place).

The first place to the right of the decimal is the place of ten cents (one-tenth of a dollar).

The second place to the right of the decimal is the place of one cents (one-hundredth of a dollar).

$ 765.43

ten cents place *(one tenth of a dollar)*
one cents place *(one hundredth of a dollar)*

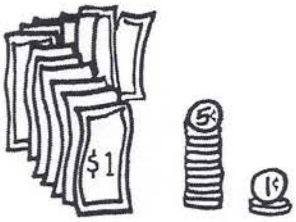


| | |
|---|---|
| $99.09 | reads *ninety-nine dollars and nine cents* |
| $9.09 | reads *nine dollars and nine cents* |
| $25.45 | reads *twenty-five dollars and forty-five cents* |
| $100.30 | reads *one hundred dollars and thirty cents* |
| $0.87 | reads *eighty-seven cents* |

There are many combinations of coins and bills that make up amounts of money:

**1** ten-dollar bill
+ **2** one-dollar bills
+ **5** dimes
+ **7** pennies

**$12.57**



**12** one-dollar bills
+ **11** nickles
+ **2** pennies

**$12.57**

**2** five-dollar bills
+ **10** quarters
+ **1** nickel
+ **2** pennies

**$12.57**

# Operations with Money

Operations with money are just like operations with decimals, because money amounts are decimals.

## Addition

Line up the decimal points carefully in both addends. Align the decimal point in the sum (answer) with the numbers above it.

On the opening weekend of a new superhero movie, kids in our city spent $53,850.50 on movie tickets. Those same kids spent $29,282.35 on candy and popcorn at the movies. How much did they spend all together?

$ 53,850.50
+$ 29,282.35
$ 83,132.85

## Subtraction

Line up the decimal points carefully in both numbers. Then, align the decimal point in the difference (answer) with the numbers above it.

All that popcorn and candy made kids really thirsty. They spent $20,554.25 on drinks at the movie. How much more did they spend on candy and popcorn than on drinks?

$ 29,282.35
-$ 20,554.25
$ 8,728.10

## Multiplication

Multiply as with whole numbers. Tally the total number of places to the right of the decimal point. Count the same number of places from the right in the product.

Anna paid the train fare for herself and four friends when they went into the city to see a movie. The round-trip fare was $5.25 for each rider. How much did Anna spend?

## Division

Move the decimal point in the divisor to make it a whole number. Move the decimal point in the dividend the same number of places. Align the decimal point in the quotient with the decimal point in the dividend. Divide as with whole numbers.

Max, a great movie-goer, spent a total of $37.60 on movie tickets last month. On the average, how much did he spend each week?

# Operations with Decimals

## Adding & Subtracting Decimals

International Falls, Minnesota is the coolest U.S. town—with average temperatures of 36.4⁰F. Key West, Florida has the warmest average, 77.7⁰F. What's the difference?

$$\begin{array}{r} 77.7^\circ \\ -\,36.4^\circ \\ \hline 41.3^\circ \end{array}$$

**Step 1:** Line up the decimal points of both numbers in the problem.

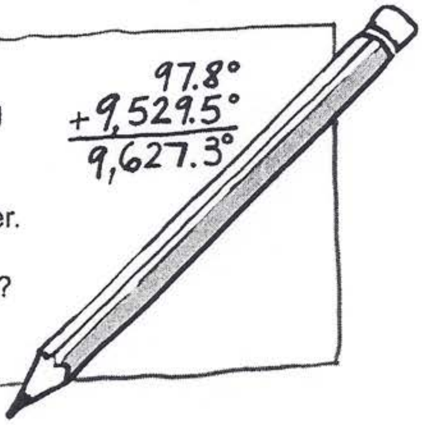
**Step 2:** Add or subtract just as with whole numbers.

**Step 3:** Align the decimal point in the sum or difference with decimal points in the numbers above.

The neighbors are complaining about the 97.8⁰ temperatures today. But the temperature on the Sun's surface is 9,529.5⁰ hotter.

What is the Sun's temperature?

$$\begin{array}{r} 97.8^\circ \\ +\,9{,}529.5^\circ \\ \hline 9{,}627.3^\circ \end{array}$$



The driest city in the U.S. is Yuma, Arizona, with 2.65 inches of precipitation yearly.

The wettest city, Quillayute, Washington, has 39.6 as much.

About how much moisture falls in Quillayute?

$$\begin{array}{r} 2.65 \\ \times\,39.6 \\ \hline 1590 \\ 2385\phantom{0} \\ +\,795\phantom{00} \\ \hline 104.940 \end{array}$$

## Multiplying Decimals

**Step 1:** Multiply as you would with whole numbers.
***Multiply 2.65 x 39.6 to get 104,940.***

**Step 2:** Count the number of places to the right of the decimal point in both factors (total).
***Count the number of places to the right of the decimal point: 2.65 has 2; 39.6 has 1, for a total of 3.***

**Step 3:** Count over from the right end of the product that same number of places.
***In the product, count 3 places backward from the right.***

**Step 4:** Insert the decimal point.
***Place the decimal point between the 4 and the 9. Quillayute's annual precipitation is about 104.94 inches.***

# Adding & Subtracting Fractions

## How to Add & Subtract Like Fractions

**Step 1:** If the fractions have like denominators, just add or subtract the numerators. (Denominators stay the same.)

**Step 2:** Reduce sums or differences to lowest terms.

$$\frac{5}{20} + \frac{3}{20} + \frac{7}{20} = \frac{15}{20} \xrightarrow{\textit{(in lowest terms)}} \frac{3}{4} \qquad \frac{8}{9} - \frac{5}{9} = \frac{3}{9} \xrightarrow[\textit{(in lowest terms)}]{} \frac{1}{3}$$

## How to Add & Subtract Unlike Fractions

**Step 1:** Find the LCM for all denominators and change the fractions to like fractions.

**Step 2:** Add or subtract the numerators. (Denominators stay the same.)

**Step 3:** Reduce sums or differences to lowest terms.

$$\frac{1}{3} + \frac{2}{8} = \frac{8}{24} + \frac{6}{24} = \frac{14}{24} \xrightarrow{\textit{(in lowest terms)}} \frac{7}{12}$$

## How to Add & Subtract Mixed Numerals

**Step 1:** Change all mixed numerals to improper fractions.

**Step 2:** Find the LCM for all the denominators and change the fractions to like fractions.

**Step 3:** Add or subtract the numerators. (Denominators stay the same.)

**Step 4:** Reduce sums or differences to lowest terms.

$$7\frac{3}{5} - 5\frac{1}{2} = \frac{38}{5} - \frac{11}{2} = \frac{76}{10} - \frac{55}{10} = \frac{21}{10} = 2\frac{1}{10}$$

# Addition

**Addition** is the combining of two or more numbers or amounts.

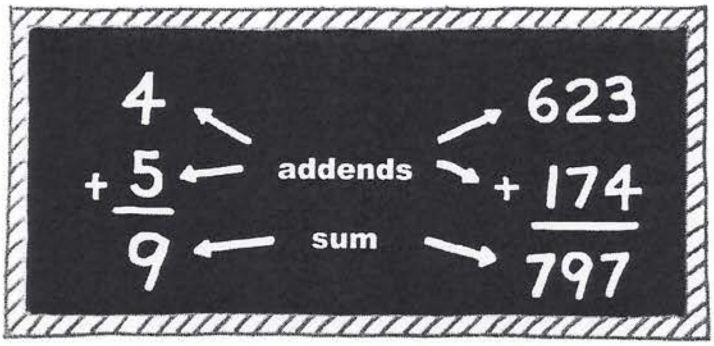
$$\underbrace{52 + 100 + 1,000 + 640}_{\text{addends}} = \underbrace{1,792}_{\text{sum}}$$

The symbol for addition is
+

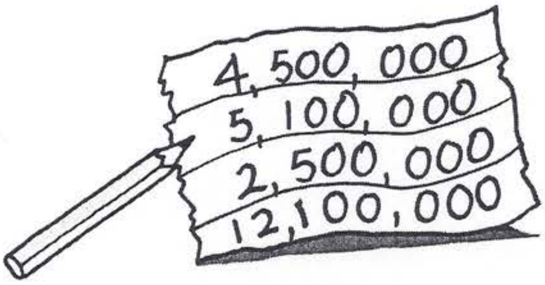
The word used for addition
is ***plus***.

The numbers being combined
are ***addends***.

The number resulting from addition
is a ***sum***.



Each year in the U.S., about 5,100,000 people go to hospital emergency rooms with pains in the stomach. About 2,500,000 visit the emergency room with head pain. Another 4,500,000 go with chest pains.

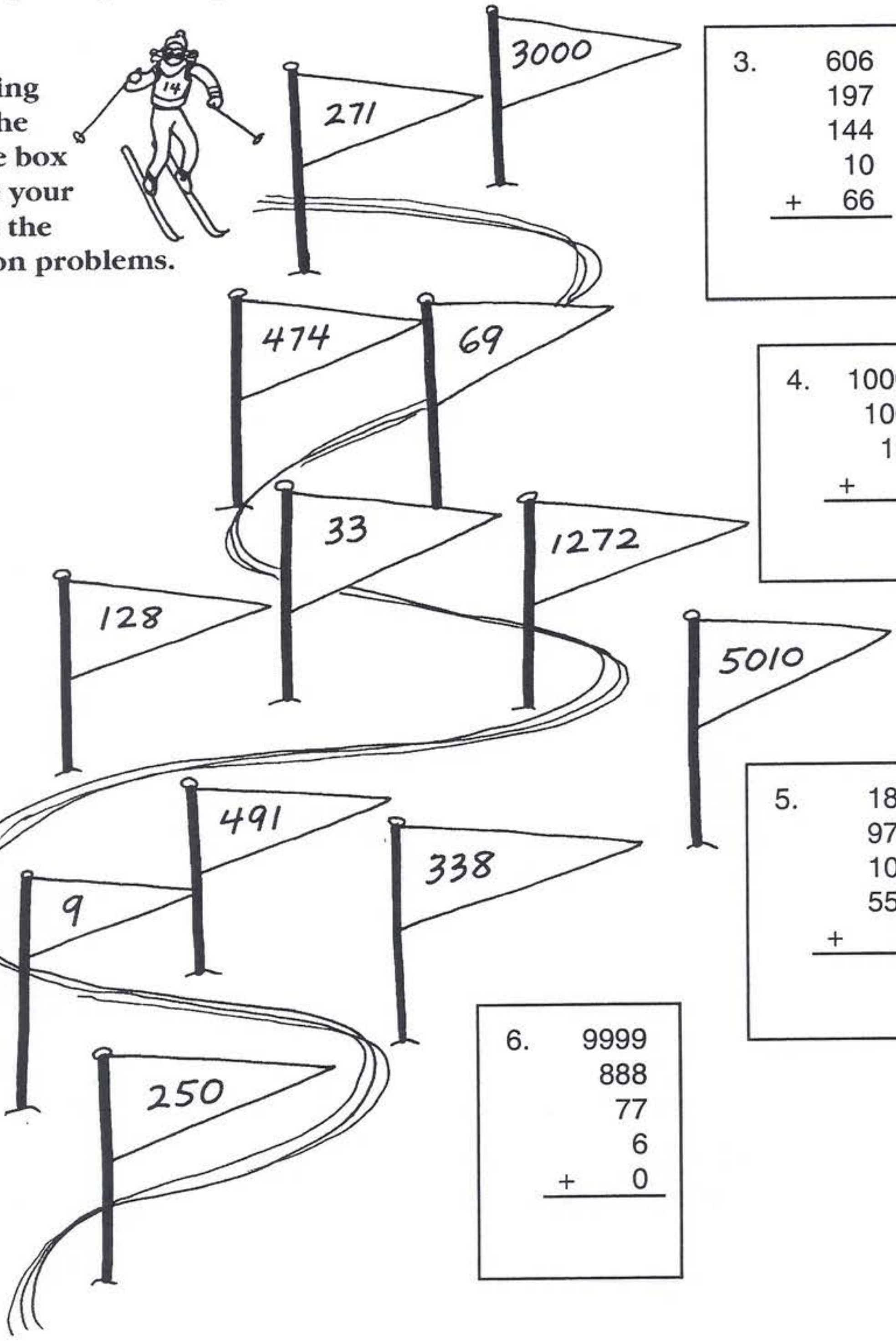


Column Addition

# HIGH SPEEDS & TOUGH TURNS

Experts say that the giant slalom takes the most technical skill of any ski event. Skiers race down the mountain over a long, steep, fast course. They must go through a series of gates marked by flags. Spectators also love to watch the downhill slalom, where skiers make high-speed turns to go through the gates at speeds of up to 80 miles per hour!

**Write a long column addition problem using all the numbers on the slalom gate flags. Use box #1 to write and solve your problem. Then solve the other column addition problems.**



1.

\+

2. 5880
999
1963
621
\+ 511

3. 606
197
144
10
\+ 66

4. 10000
1000
100
\+ 11

5. 182
974
102
555
\+ 3

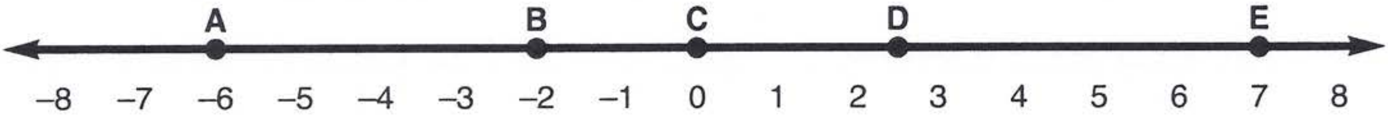
6. 9999
888
77
6
\+ 0

# Integers

The set of **integers** is the set of numbers including zero and all numbers greater than or less than zero.

**Positive integers** are greater than zero.

**Negative integers** are less than zero.



The number line is a graph of some points corresponding to integers.

The location of each point is called the **coordinate**.

**A** is the graph of negative 6.
Its coordinate is –6

**B** is the graph of negative 2.
Its coordinate is –2.

**D** is the graph of positive 2.5.
Its coordinate is 2.5.

**E** is the graph of positive 7.
Its coordinate is 7.

## Opposites

Every integer has an opposite.
Any two numbers that are the same distance from zero are opposites.

**–10 is the opposite of 10**

**33.5 is the opposite of –33.5**

**$\frac{1}{2}$ is the opposite of $-\frac{1}{2}$**

## Absolute Value

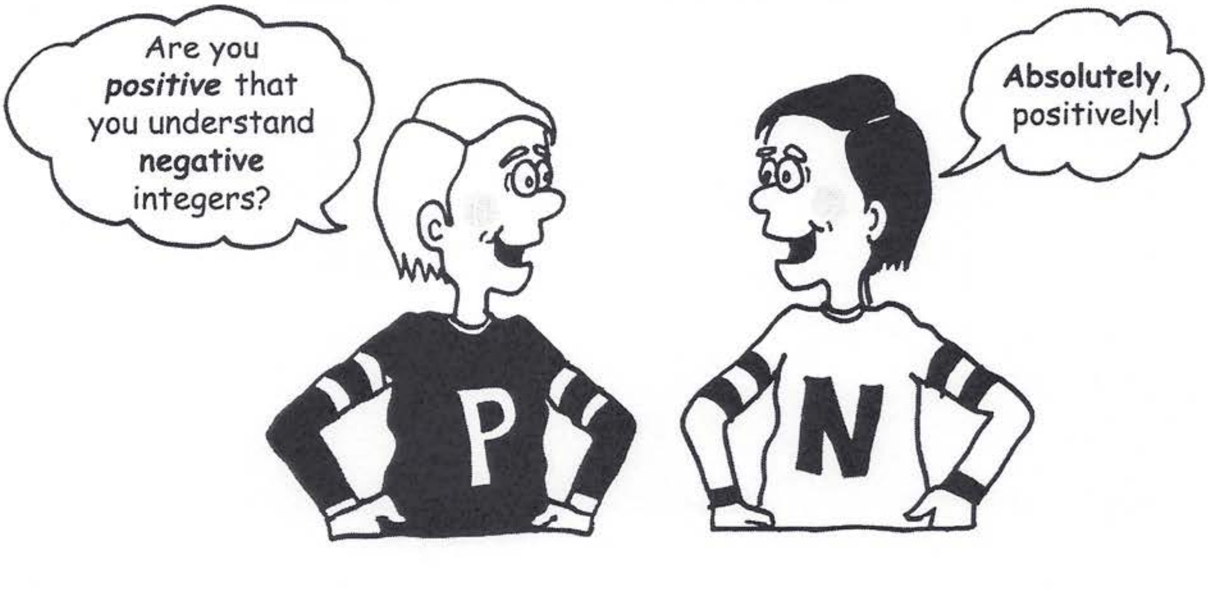
The absolute value of a number is its distance from zero on a number line.

| | is the symbol for absolute value.

| 8 | reads ***the absolute value of 8*****.**

**–25 and 25 have the same absolute value.**

**$|-15| = 15$** **$|15| = 15$**



# Know Your Properties

### Commutative Property for Addition:

The order in which numbers are added does not affect the sum.

Examples:
$$7 + 11 = 11 + 7$$
$$255 + 144 = 144 + 255$$

### Commutative Property for Multiplication:

The order in which numbers are multiplied does not affect the product.

Examples:
$$5 \times 9 = 9 \times 5$$
$$13 \times 10 = 10 \times 13$$

### Associative Property for Addition:

The way in which numbers are grouped does not affect the sum.

Examples:
$$(4 + 3) + 6 = 4 + (3 + 6)$$
$$(1{,}010 + 584) + 36 = 1{,}010 + (584 + 36)$$

### Associative Property For Multiplication:

The way in which numbers are grouped does not affect the product.

Examples:
$$(5 \times 2) \times 4 = 5 \times (2 \times 4)$$
$$50 \times (30 \times 200) = (50 \times 30) \times 200$$

### Distributive Property:

To multiply a sum of numbers, you may first add the numbers in parentheses and then multiply the sum.

Examples:
$$4 \times (6 + 3) = 4 \times (9) = 36$$
$$9 \times (10 + 5) = 9 \times 15 = 135$$

**OR,** you may first multiply the addends separately, then add the products.

Examples:
$$\begin{aligned} 4 \times (6 + 3) &= (4 \times 6) + (4 \times 3) \\ &= 24 + 12 \\ &= 36 \end{aligned}$$
$$\begin{aligned} 9 \times (10 + 5) &= (9 \times 10) + (9 \times 5) \\ &= 90 + 45 \\ &= 135 \end{aligned}$$

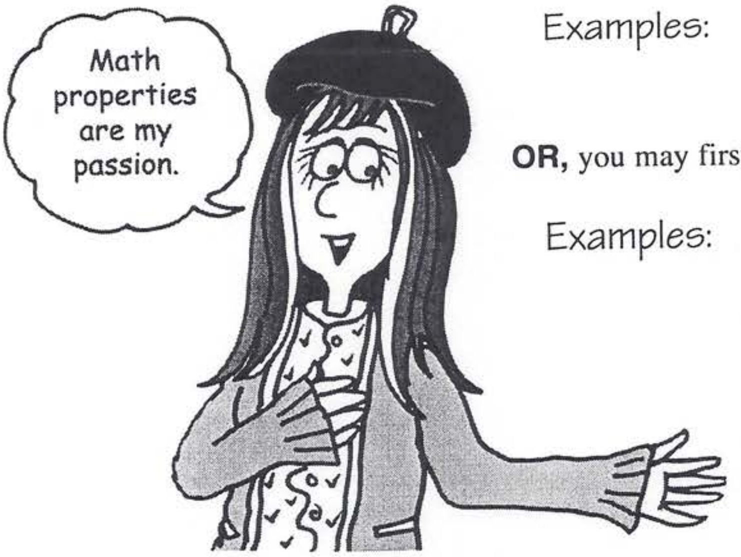
**Ask**

**Dr. Cal Q. Layter**

*What's so important about properties, anyway?*

**Answer:**
**Properties are rules that numbers follow in operations.**

- **Knowing the properties will help you understand how numbers work.**
- **Knowing the properties will help you find the correct solutions to many problems.**



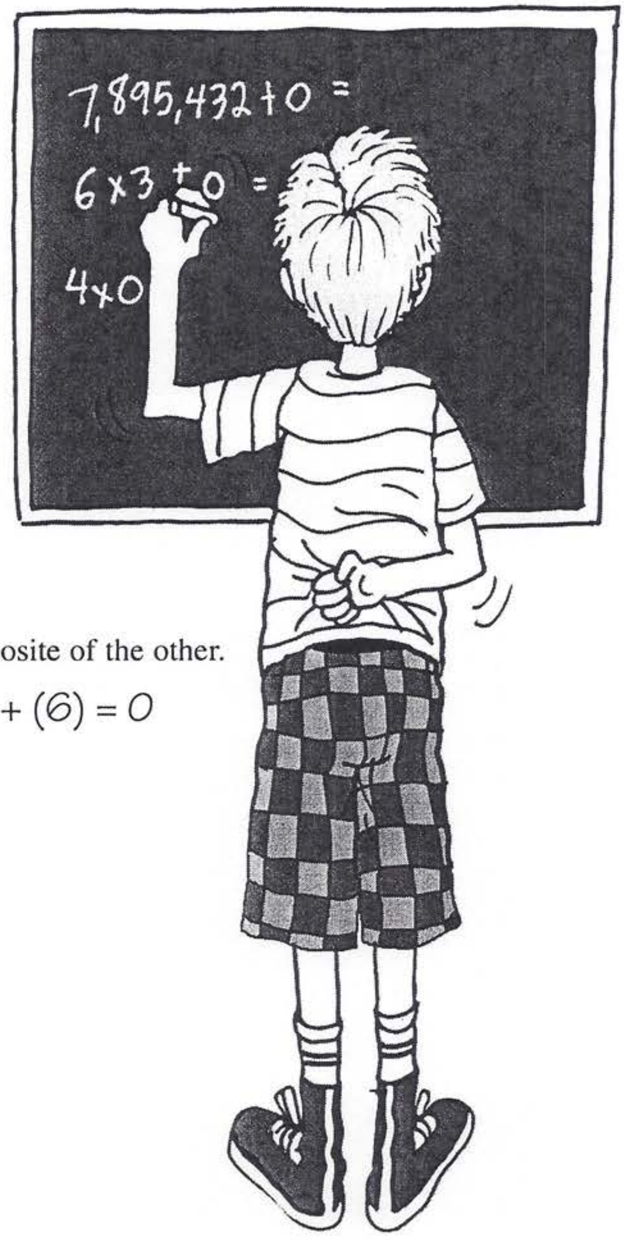
### Identity Property For Addition:

The sum of zero and any number is that number.

Examples: $9 + 0 = 9$

$486 + 0 = 486$

$0 + 68{,}117 = 68{,}117$



### Identity Property For Multiplication:

The product of 1 and any number is that number.

Examples: $6 \times 1 = 6$

$1 \times 25 = 25$

$7{,}993 \times 1 = 7{,}993$

### Opposites Property:

If the sum of two numbers is 0, then each number is the opposite of the other.

Examples: −6 is the opposite of 6 because $-6 + (6) = 0$

−42 is the opposite of 42 because $-42 + (42) = 0$

### Zero Property for Addition:

The sum of zero and any number is that number.

Examples: $0 + 8 = 8$

$407 + 0 = 407$

### Zero Property for Multiplication:

The product of zero and any number is zero.

Examples: $0 \times 8 = 0$

$6 \times 0 = 0$

$5{,}000 \times 0 = 0$

### Equation Properties:

When adding or subtracting the same number or multiplying or dividing by the same number on both sides of an equation, the result is still an equation.

Examples: $n - 8 = 7,$

$n - 8\ (+\ 8) = 7\ (+\ 8)$

$n = 15$

$5n = 30$

$5n\ (\div\ 5) = 30\ (\div\ 5)$

$n = 6$

# Finding the Missing Operations in Word Problems

- Watch for these words in problems or directions. They are signals for ***addition***!

*sum*
*together*
*all together*
*average*
*total*
*both*
*increased by*

- Watch for these words in problems or directions. They are signals for ***subtraction***!

*difference*
*left over*
*take away*
*fewer than*
*much less*
*less than*
*remain*
*have left*
*much more*
*reduced by*

- Watch for these words in problems or directions. They are signals for ***multiplication***!

*times*
*how many times*
*a product of*
*twice as much as*

- Watch for these words in problems or directions They are signals for ***division***!

*divided by*
*average*
*half as much*
*sharing*
*equal parts*
*any fraction*

In the past five years, 57 injuries sent members of the Tioga Tumblers gymnastics team to the emergency room. This year, there have been 9 injuries requiring emergency room visits. What is the average number per year over the six years?

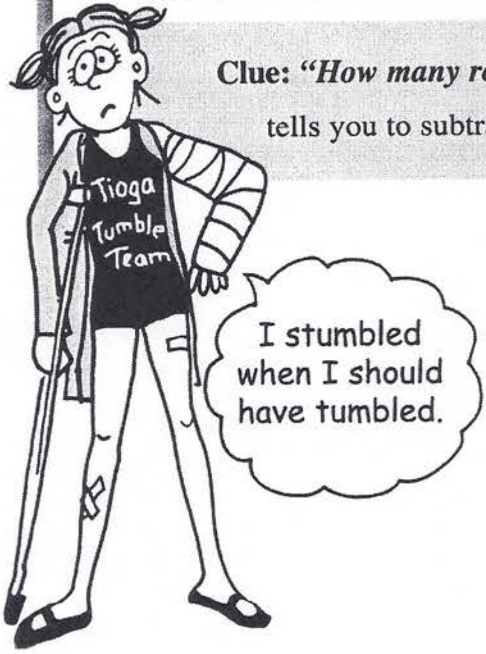
**Clue:** ***"What is the average"*** tells you to add and divide.

Roxy had four times as many injuries as Maxie, who had 7. How many injuries did Roxy have?

**Clue:** ***"four times as many"*** tells you to multiply.

The team has 38 members. This year, 19 have suffered injuries. How many team members remain uninjured?

**Clue:** ***"How many remain"*** tells you to subtract.



# Division

**Division** is repeated subtraction. When you divide, you are subtracting the same number over and over again.

You can subtract 8 from 48 six times: 48 – 8 – 8 – 8 – 8 – 8 – 8 = 0

Or, you can divide 48 ÷ 8 and get 6.

48 ÷ 8 = 6 means there are 6 groups of 8 in 48.

The symbol for division is $\overline{)\phantom{0}}$ or ÷.

**Division** is a way of finding out how many times one number (the **divisor**) will fit into another number (the **dividend**).

The number resulting from division is the **quotient**.

If a divisor does not fit an even number of times into a dividend, there will be a number left over. This is called the **remainder.**

$8 \leftarrow$ *quotient*

*divisor* $\rightarrow 9\overline{)72} \leftarrow$ *dividend*

$$\underset{\substack{\uparrow\\ \textit{dividend}}}{9{,}633} \div \underset{\substack{\uparrow\\ \textit{divisor}}}{3} = \underset{\substack{\uparrow\\ \textit{quotient}}}{3{,}211}$$

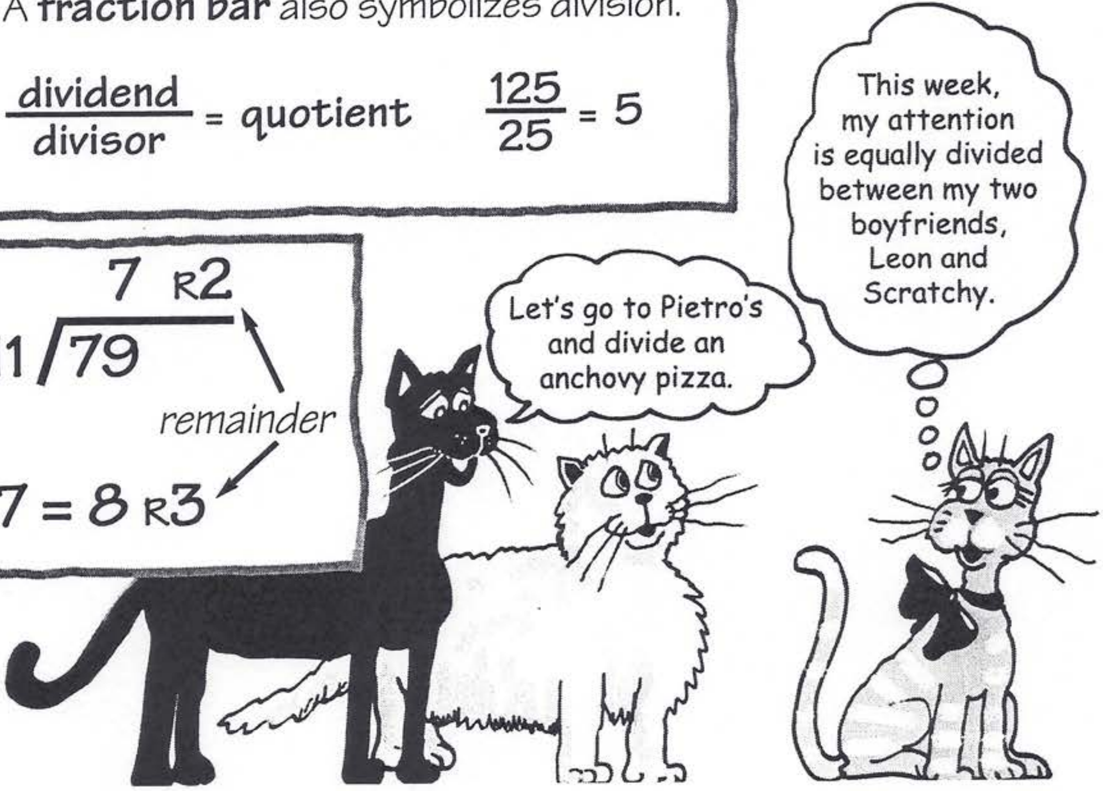
A **fraction bar** also symbolizes division.

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} \qquad \frac{125}{25} = 5$$

$$\begin{array}{r} 7\ \text{R}2 \\ 11\overline{)79} \end{array}$$

*remainder*

$59 \div 7 = 8\ \text{R}3$



# Division with One-Digit Divisors

$$\begin{array}{r} 159\ \text{R}\ 2 \\ 5\,\overline{)797} \\ -5 \\ \hline 29 \\ -25 \\ \hline 47 \\ -45 \\ \hline 2 \end{array}$$

**Step 1: Does 5 go into 7? (yes—1 time)**

Write the 1 above the 7.
Multiply 1 x 5. Write the product under the 7.
Subtract 7 – 5 ( = 2).
Bring the next digit (9) down next to the 2.

**Step 2: Does 5 go into 29? (yes—5 times)**

Write the 5 above the 9.
Multiply 5 x 5. Write the product under 29.
Subtract 29–25 ( = 4).
Bring the next digit (7) down next to the 4.

**Step 3: Does 5 go into 47? (yes—9 times)**

Write the 9 above the 7.
Multiply 9 x 5. Write the product under 47.
Subtract 47 – 45 ( = 2).
Write the remainder (2) next to the quotient.

# Division with Multiples of Ten

$$36{,}000 \div 100 =$$

**Step 1:**
Place a decimal point
after the dividend:
36,000.

**Step 2:**
Move the decimal point
one place to the left
for each zero in the divisor:
360.00
*(The decimal point was moved*
*2 places because 100 has 2 zeros.)*

**Step 3:**
Drop any zeros to the
right of the decimal point:
$36{,}000 \div 100 = 360$

# Division with Larger Divisors

When the divisor has more than one digit, division problems can get very tricky.

Here are some steps to help you handle this process without feeling baffled.

```
     370 R 8
32 ) 11,848
    -96
     224
    -224
      08
     - 0
       8
```

**Step 1: Does 32 go into 1? (no)**

**Step 2: Does 32 go into 11? (no)**

**Step 3: Does 32 go into 118? (yes)**

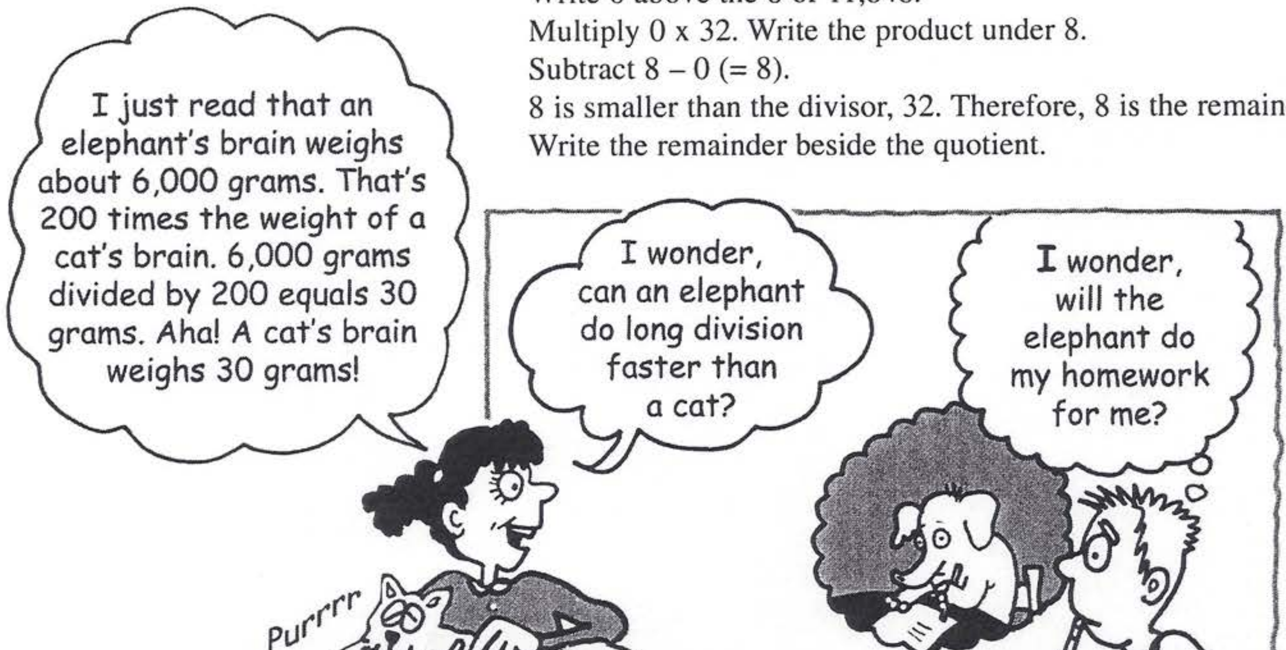
Round 32 to the closest 10. (30)
Estimate the number of 30s in 118. *(about 3)*
Write 3 above the 8 of 118.
Multiply 3 x 32. Write the product under 118.
Subtract 118 – 96 (= 22).
Bring down the next digit (4) beside the 22.

**Step 4: Does 32 go into 224? (yes)**

Round 32 to 30 again.
Estimate the number of 30s in 224. *(about 7)*
Write 7 above the 4 of 1,184.
Multiply 7 x 32. Write the product under the 224.
Subtract 224 – 224 (= 0).
Bring down the next digit (8) beside the 0.

**Step 5: Does 34 go into 8? (no)**

Write 0 above the 8 of 11,848.
Multiply 0 x 32. Write the product under 8.
Subtract 8 – 0 (= 8).
8 is smaller than the divisor, 32. Therefore, 8 is the remainder.
Write the remainder beside the quotient.



## Estimate

An **estimate** is an approximate solution to a problem. Sometimes, a problem does not need an exact answer, and an estimation is a quick or practical solution.

**Get Sharp Tip #33**

**Don't forget to use rounding when you estimate. It's a great estimation tool.**

### *The Problem:*

The 34-member bowling team spends plenty of money while they practice for their tournaments. They have 13 practices a week during their 8-week training camp. Each practice session costs $9 per team member.

The bowlers and their families raised $25,000 at fundraising events.

**Will that be enough to pay for the practice sessions?**

***Round the 34 players to 30.***

***Round the 13 practices to 10.***

***Round the 8 weeks of the camp to 10.***

***Multiply 30 x 10 x 10 x 10 to get $30,000.***

## Extend a Pattern

| Game | Louie's Scores |
|---|---|
| 1 | 156 |
| 2 | 162 |
| 3 | 168 |
| 4 | 174 |
| 5 | 180 |
| 6 | 186 |
| 7 | 192 |
| 8 | |
| 9 | |
| 10 | |

Sometimes the best strategy is to look for a pattern in the data. Then extend (continue) the pattern to find a solution to the problem.

### *The Problem:*

The table shows Louie's scores in the first 7 games of the bowling tournament.

**What do you predict will be his score in the next three games?**

Notice the pattern: Louie's score increased by 6 points from each game to the next.

***Extend the pattern to find his score in games 8, 9, and 10. (He will score 198, 204, and 210!)***

### Dividing a Number by Itself

**ANY** number divided by itself yields **1**!

$$65 \div 65 = 1$$

No matter HOW BIG the number is, the quotient is still ONE.

$$999{,}999{,}999 \div 999{,}999{,}999 = 1$$

### Dividing a Number by One

**ANY** number divided by 1 yields that number.

$$95 \div 1 = 95$$

$$700{,}000 \div 1 = 700{,}000$$

## Divisibility

A number is ***divisible*** by another number if the quotient of the two numbers is a whole number. (50 is divisible by 5 because the quotient is a whole number, 10.)

A number is ***divisible by 2*** if the last digit is 0, 2, 4, 6, or 8.

A number is ***divisible by 3*** if the sum of its digits is divisible by 3.

A number is ***divisible by 4*** if the last two digits are divisible by 4.

A number is ***divisible by 5*** if the last digit is 0 or 5.

A number is ***divisible by 6*** if the number is divisible by both 2 and 3.

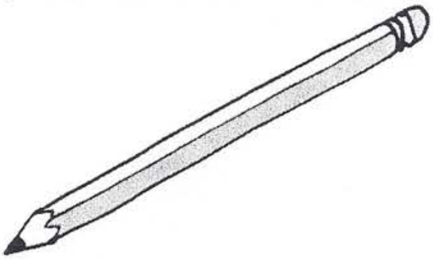
A number is ***divisible by 8*** if the last three digits are divisible by 8.

A number is ***divisible by 9*** if the sum of its digits is divisible by 9.

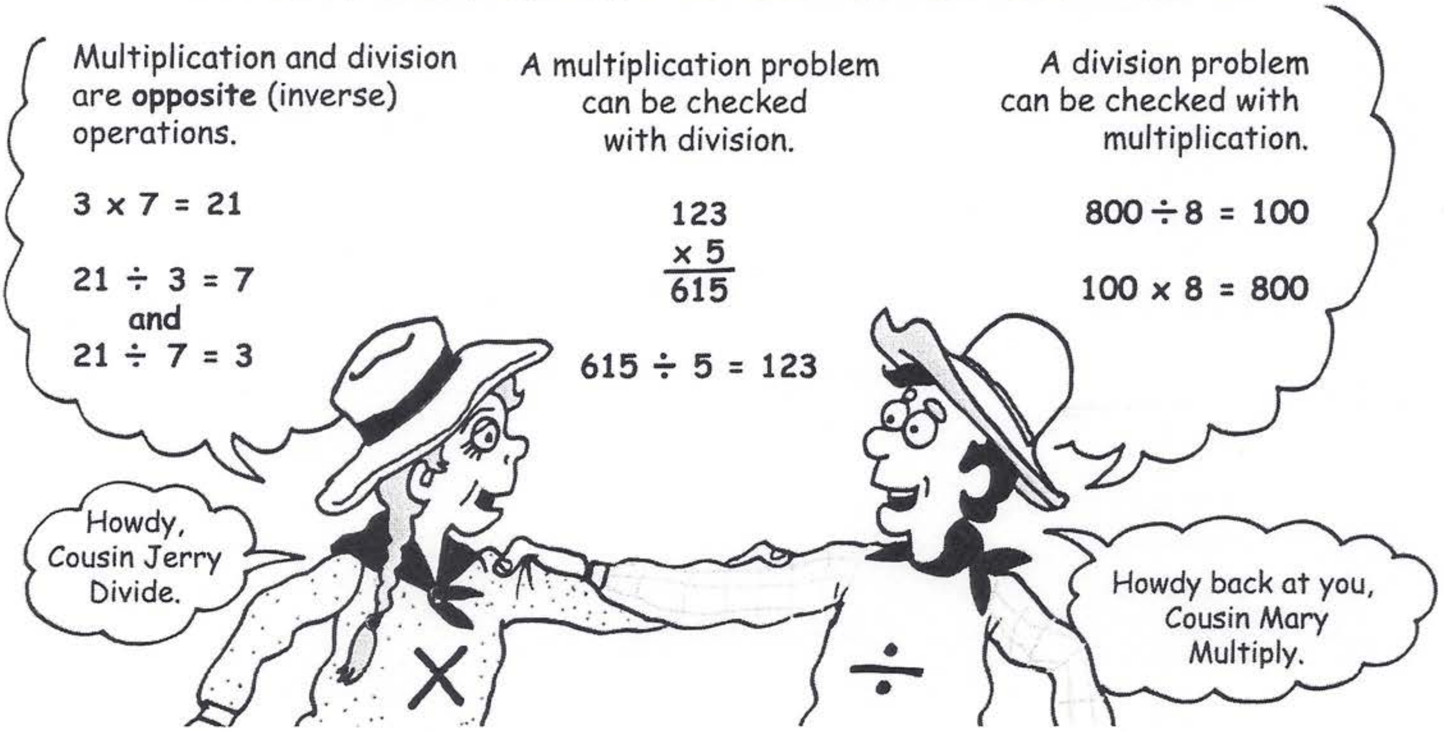
A number is ***divisible by 10*** if the last digit is 0.

**Get Sharp Tip #10**

There is **no** division by zero. It's impossible.



## MULTIPLICATION AND DIVISION ARE RELATIVES



Multiplication and division are **opposite** (inverse) operations.

3 x 7 = 21

21 ÷ 3 = 7

and

21 ÷ 7 = 3

A multiplication problem can be checked with division.

$$\begin{array}{r} 123 \\ \times\ 5 \\ \hline 615 \end{array}$$

615 ÷ 5 = 123

A division problem can be checked with multiplication.

800 ÷ 8 = 100

100 x 8 = 800

# Multiplication

**Multiplication** is repeated addition. When you multiply, you are adding the same number over and over again.

The symbol for multiplication is

**× or • .**

The word used for multiplication is **times.**

The numbers being multiplied are **factors.**

The number resulting from multiplication is a **product.**

You can add 6 + 6 + 6 + 6 + 6 + 6 + 6 to get 42.

Or, you can multiply 6 x 7 and get 42.

6 x 7 means seven groups of six.

$$\begin{array}{rcr} 6 & \leftarrow \text{factors} \rightarrow & 111 \\ \underline{\times\ 7} & & \underline{\times\ \ 4} \\ 42 & \leftarrow \text{product} \rightarrow & 444 \end{array}$$

$$\underset{\text{factor}}{3{,}333} \times \underset{\text{factor}}{3} = \underset{\text{product}}{9{,}999}$$

### Multiplying by One

**ANY** number multiplied by one has a product the same as the number!

65 x 1 = 65

999,999 x 1 = 999,999

### Multiplying by Zero

**ANY** number multiplied by zero is **0**!

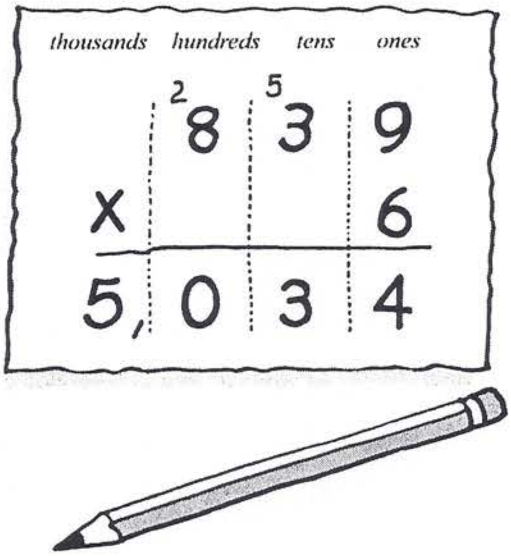
65 x 0 = 0

No matter HOW BIG the number is, the product is still ZERO.

0 x 999,999,999,999 = 0

# Multiplication with Renaming

Sometimes you will need to **rename** (or regroup) numbers to complete a multiplication task. Here's how it works.



**Step 1: Multiply the ones. 6 x 9 = 54 ones.**
Rename the 54 ones as 5 tens and 4 ones.
Write the 5 above the tens column, and the 4 in the ones place in the product.

**Step 2: Multiply the tens: 6 x 3 = 18.**
Add the 5 tens. 18 + 5 = 23 tens.
Rename the 23 tens as 2 hundreds and 3 tens.
Write the 2 above the hundreds column, and the 3 in the tens place in the product.

**Step 3: Multiply the hundreds: 6 x 8 = 48 hundreds.**
Add the 2 hundreds: 48 + 2 = 50 hundreds
Rename the 50 hundreds as 5 thousands and 0 hundreds.
Write the 0 in the hundreds place, and the 5 in the thousands place in the product.

---

## Multiplication with Multiples of Ten

30 x 800 =

**Step 1:**

Drop the zeros and rewrite the problem as a basic multiplication fact.

3 x 8

**Step 2:**

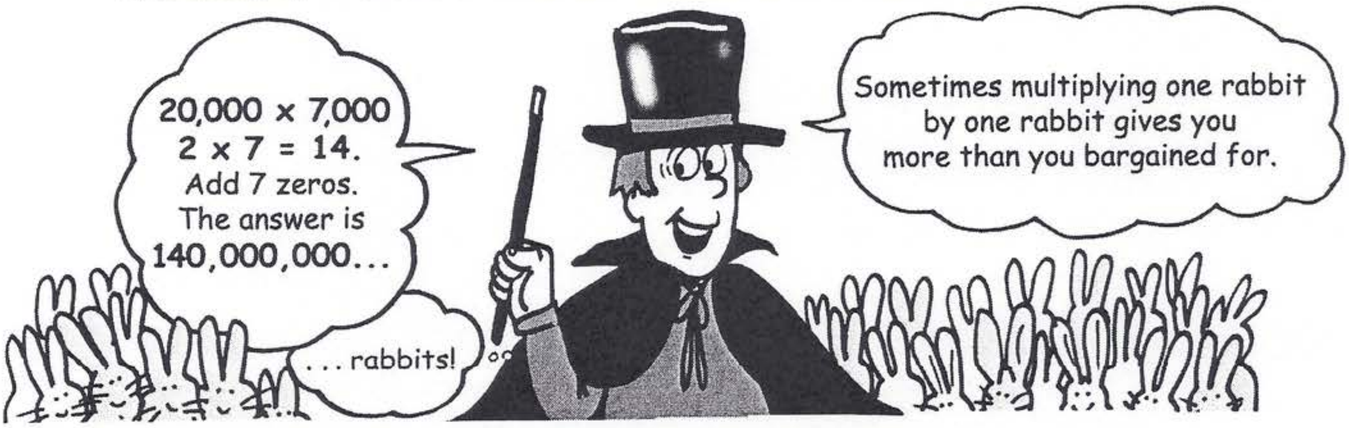
Find the product.

3 x 8 = 24

**Step 3:**

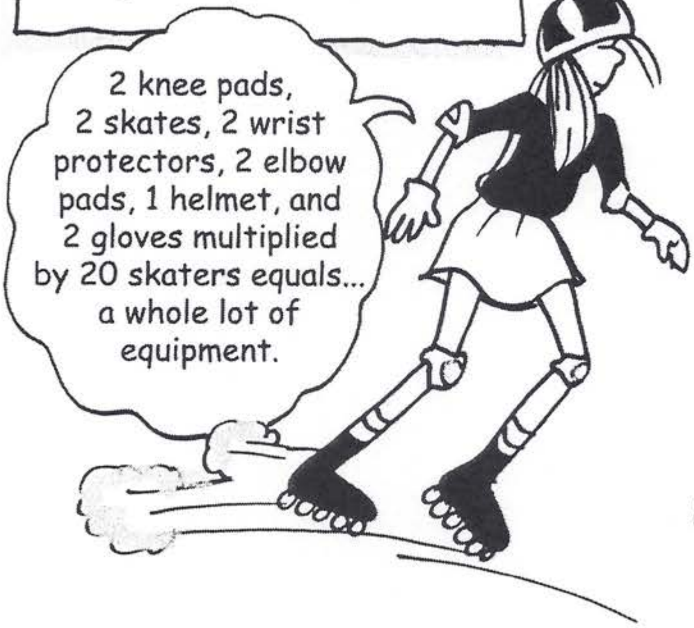
At the end of the product, write the same number of zeros you dropped.

24,000



# Multiplication by Larger Numbers

| ten thousands | thousands | hundreds | tens | ones |
|---|---|---|---|---|
| | | 3 | 6 | 8 |
| | x | 2 | 5 | 7 |
| | 2 | 5 | 7 | 6 |
| 1 | 8 | 4 | 0 | |
| +7 | 3 | 6 | | |
| 9 | 4, | 5 | 7 | 6 |



**Get Sharp Tip #9**

There are 100 basic multiplication facts with factors 1-10. If you learn the first 55, you will know all 100—because the order of the factors does not change the product.

## Step 1: Multiply by ones.

**Multiply 7 x 8 (7 x 8 = 56 ones)**
Rename the 56 ones as 5 tens and 6 ones.

**Multiply 7 x 6 (7 x 6 = 42 tens).**
Add the 5 tens. (42 + 5 = 47 tens)
Rename the 47 tens as 4 hundreds and 7 tens.

**Multiply 7 x 3 (7 x 3 = 21 hundreds).**
Add the 4 hundreds. (21 + 4 = 25 hundreds)
Rename the 25 hundreds as 2 thousands and 5 hundreds.

## Step 2: Multiply by tens.

**Multiply 5 x 8 (5 x 8 = 40 tens)**
Rename the 40 tens as 4 hundreds and 0 tens.

**Multiply 5 x 6 (5 x 6 = 30 hundreds).**
Add the 4 hundreds. (30 + 4 = 34 hundreds)
Rename the 34 hundreds as 3 thousands and 4 hundreds.

**Multiply 5 x 3 (5 x 3 = 15 thousands).**
Add the 3 thousands (15 + 3 = 18 thousands).
Rename the 18 thousands as 1 ten thousand and 8 thousands.

## Step 3: Multiply by hundreds.

**Multiply 2 x 8 (2 x 8 = 16 hundreds)**
Rename the 16 hundreds as 1 thousand and 6 hundreds.

**Multiply 2 x 6 (2 x 6 = 12 thousands).**
Add the 1 thousand. (12 + 1 = 13 thousand)
Rename the 13 thousands as 1 ten thousand and 3 thousands.

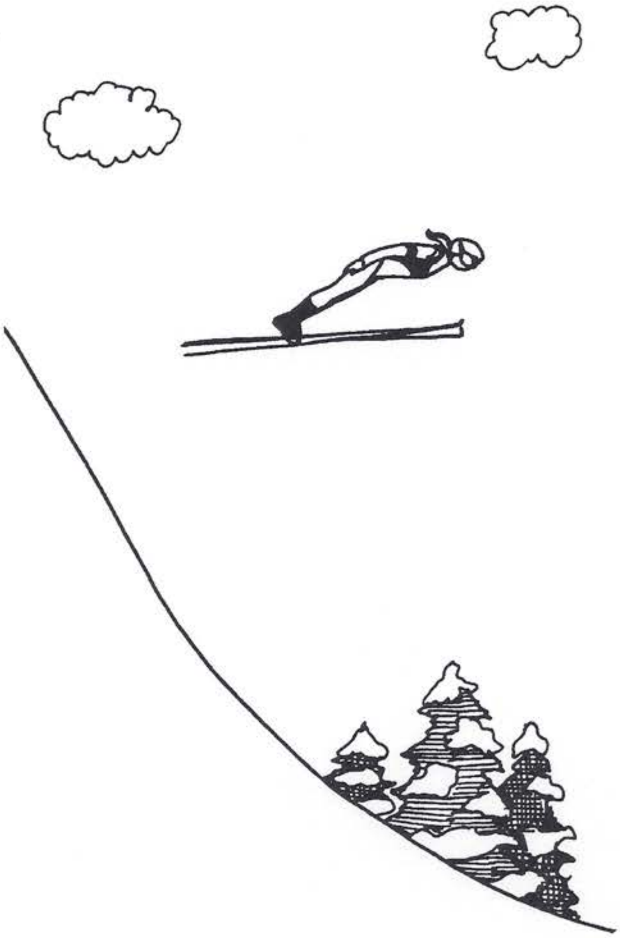
**Multiply 2 x 3 (2 x 3 = 6 ten thousands).**
Add the 1 ten thousand. (6 + 1 = 7 ten thousands.)

## Step 4: Add the columns.

Multiply & Divide with Multiples of 10

# SAILING WITHOUT A SAIL

It's spectacular . . . breathtaking . . . awesome! Crowds at the Winter Olympics always love to watch the ski jumpers sailing through the air. Spectators hold their breath until the skier lands safely on the ground! Skiers gain points for strong take-offs, smooth flights, clean landings, and distance. Skiers take off into the air from jumps as high as 120 meters and sail for hundreds of feet.



**Use multiplication to figure out these distances.**

1. 23 meters x 10 = ____________
2. 23 meters x 100 = ____________
3. 31 meters x 30 = ____________
4. 111 meters x 1,000 = ____________
5. 505 meters x 10 = ____________
6. 2,222 meters x 400 = ____________
7. 717 meters x 10,000 = ____________
8. 4,024 meters x 20 = ____________
9. 70 meters x 40 = ____________
10. 250 meters x 1,000 = ____________

**Use division to figure out these distances.**

11. 4,400 meters ÷ 10 = ____________
12. 4,400 meters ÷ 100 = ____________
13. 4,400 meters ÷ 200 = ____________
14. 1,000 meters ÷ 10 = ____________
15. 1,000 meters ÷ 100 = ____________
16. 330 meters ÷ 10 = ____________
17. 880,000 meters ÷ 1,000 = ____________
18. 700 meters ÷ 70 = ____________
19. 5,600 meters ÷ 80 = ____________
20. 61,070 meters ÷ 10 = ____________

**Olympic Fact**

*Judges stand at one-meter intervals along the edge of the hill and watch to see where the ski jumpers land. They decide the distances with their eyes instead of measuring with any tools.*

# Subtraction

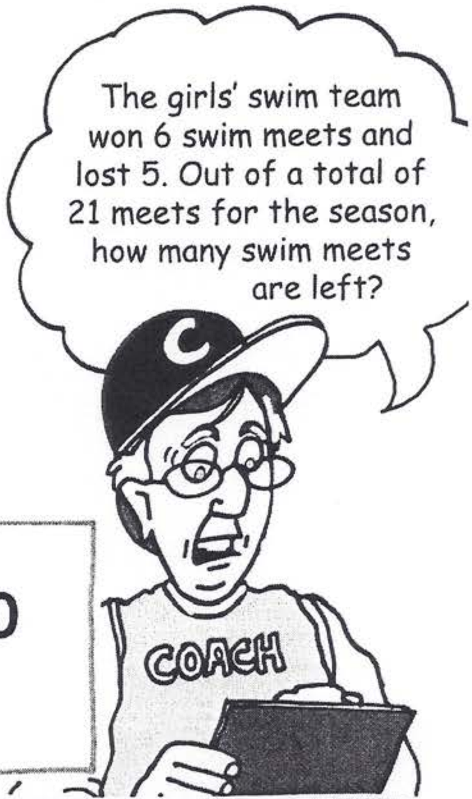
**Subtraction** is the operation of finding a missing addend (or, the taking away of one number or amount from another).

The symbol for subtraction is **—**

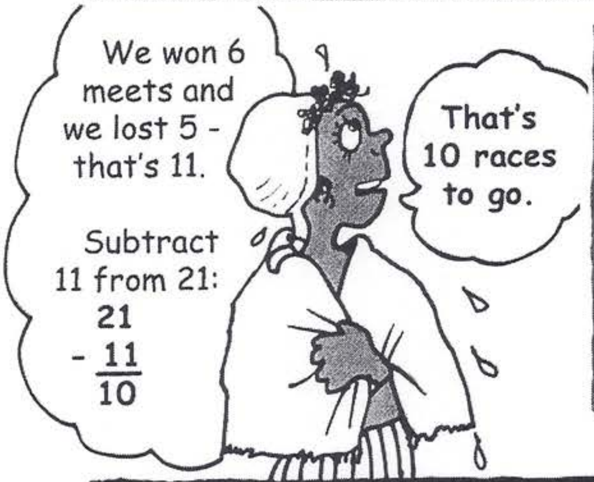
The word used for addition is ***minus***.

The number being subtracted from is the ***minuend***.

The number being subtracted is the ***subtrahend***.



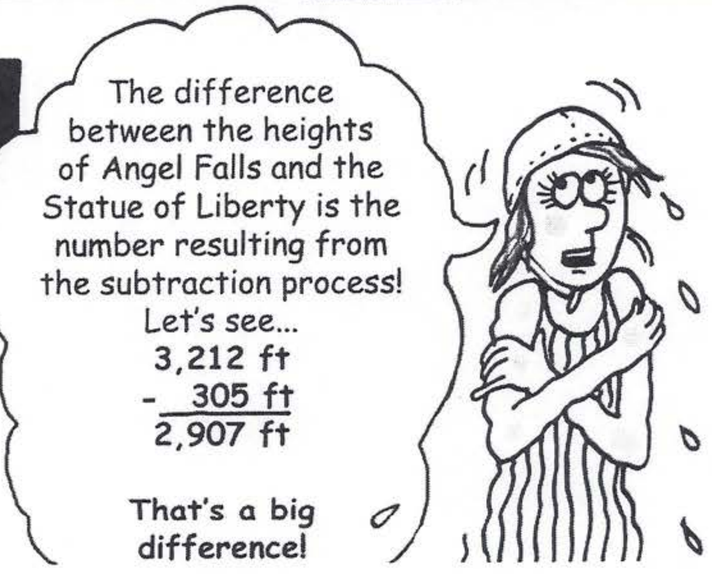
$$\underset{\text{minuend}}{\underset{\uparrow}{505,000}} - \underset{\text{subtrahend}}{\underset{\uparrow}{5,000}} = \underset{\text{difference}}{\underset{\uparrow}{500,000}}$$



| | | |
|---|---|---|
| 8 ← | minuend → | 988 |
| - 3 ← | subtrahend → | - 237 |
| 5 ← | difference → | 751 |

## WHATS THE DIFFERENCE?

Angel Falls in Venezuela is the world's tallest waterfall. It is 3,212 feet tall. The Statue of Liberty is 305 feet tall. What's the difference between their heights?



# Subtraction with Borrowing (or Renaming)

Sometimes a digit in the minuend is smaller than the digit of the same place in the subtrahend. When this happens, it is necessary to ***borrow*** from the next place to the left.

***Borrowing*** is the same as ***renaming***. It means exchanging a ten to make a number in the ones place **larger** than the digit in the subtrahend. (OR, it might mean exchanging a hundred for 10 tens, or a thousand for 10 hundreds, etc.)

### Addition & Subtraction Are Relatives!

Addition and subtraction are opposite (inverse) operations.

$9 + 7 = 16$

$16 - 9 = 7$

and $16 - 7 = 9$

An addition problem can be checked with subtraction.

$$\begin{array}{r} 8{,}222 \\ +\,9{,}666 \\ \hline 17{,}888 \end{array} \qquad \begin{array}{r} 17{,}888 \\ -\,9{,}666 \\ \hline 8{,}222 \end{array}$$

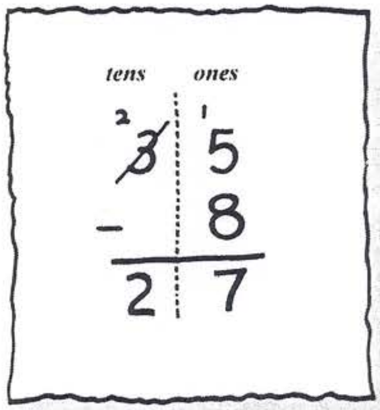
A subtraction problem can be checked with addition.

$$\begin{array}{r} 50{,}000 \\ -\quad 500 \\ \hline 49{,}500 \end{array} \qquad \begin{array}{r} 49{,}500 \\ +\quad 500 \\ \hline 50{,}000 \end{array}$$

In this example, 8 is too large to subtract from 5. So, one of the tens is ***borrowed or renamed*** as 10 ones.

Now there are 15 ones.
8 can be subtracted from 15.

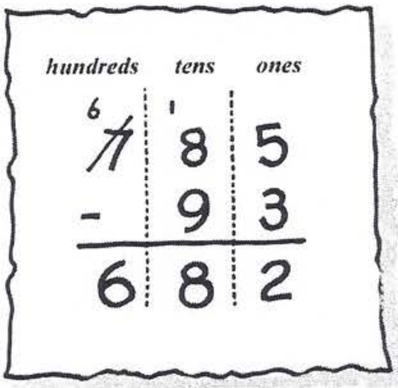
That leaves only 2 tens.
(See the 2 written above the tens place.)



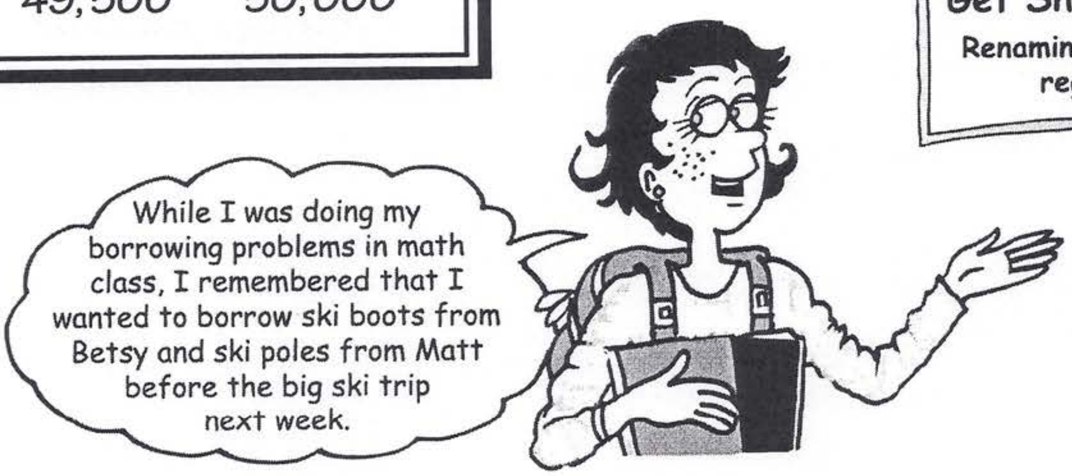
In this, the 8 in the tens place is smaller than the 9 in the tens place. So, one of the hundreds is ***borrowed or renamed*** as 10 tens.

Now there are 18 tens.
9 can easily be subtracted from 18.

This leaves only 6 hundreds.
(See the 6 written above the hundreds place.)



**Get Sharp Tip #8**
Renaming is also called regrouping.



Subtract Whole Numbers

# KNOCK OUT!

Boxing was not allowed at the first modern Olympics in 1896 because it was considered too ungentlemanly and dangerous. Today it is a very popular Olympic sport. Some of the world's greatest boxers, such as Floyd Patterson, Muhammad Ali, Sugar Ray Leonard, Joe Frazier, Leon Spinks, and Evander Holyfield, won Olympic medals before becoming professional boxers.

**See if you can knock out these subtraction problems by getting all the answers right!**

1. 500
   − 229

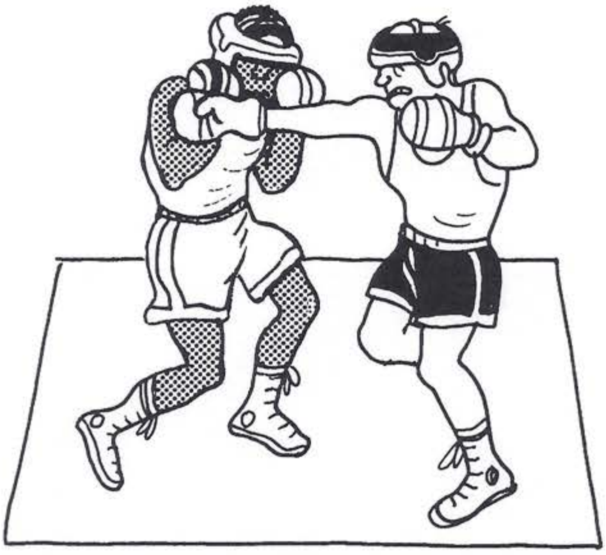
2. 900
   − 683

3. 40
   − 26

4. 300
   − 258

5. 407
   − 133

6. 90
   − 55



7. 800
   − 393

8. 5500
   − 203

9. 9050
   − 5348

10. 7001
    − 6420

11. 6110
    − 456

12. 8006
    − 731

13. 800,321
    − 79,001

14. 32,000
    − 19,862

**Olympic Fact**

One of the most memorable moments of the 1996 Summer Olympic Games in Atlanta was when boxing legend and 1960 gold medal-winner Muhammad Ali (who suffers from Parkinson's disease) lit the Olympic torch.